A Spring-Dashpot-String Element for Modeling Spinal Column Dynamics

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Abstract

A new dynamic element is introduced to model the motion of the spine. This element will allow easily modeling of the flexion, extension and bending motions of human spine with a minimum number of degrees of freedom. In this paper, we present the equations of the new element and the imposed constraint to guarantee a realistic spinal motion.

1. Introduction

Biomechanics is playing an important role in human life. Human spine is given particular attention, due to its frequent involvement in accident-induced injuries and almost widespread appearance in common diseases such as low back pain. The inter-vertebral motion, although basically understood, still poses many open problems. This is so because pairs of vertebrae undergo in general six-dimensional motion relative to one another and display a high degree of coupling between gross translational and rotational degrees-of-freedom due to restraints imposed by ligaments and muscles and the nature of the inter-vertebral disks. In this paper we propose a simple and fast modeling technique for spinal flexion, extension and bending motions. The spine is modeled as dynamic elements which we call *spinal elements*. Each spinal element consists of two masses, two springs, a dashpot and a string. These elements are connected together with high stiffness springs at the outer masses (vertebras) to satisfy the rigidity condition. The values of the spring and damper coefficients could be estimated using regression techniques from the recorded spine motion data. If the resulting model is not realistic, more elements could be added incrementally. The parameters are re-estimated and the procedure continues until we have good agreement with recorded data. In the following sections, we present the equations of the new element and the imposed constraint to guarantee a realistic spinal motion.

2. Spinal Element

The spinal element consists of a longitudinal piece cut thought two adjacent vertebras, as shown in figure (1a), with the disk, ligaments and muscles are all included in the model.

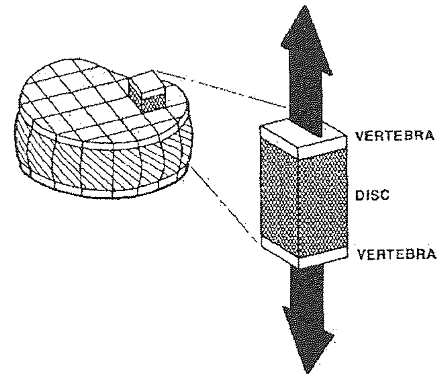


Figure (1a) Spinal Element (ligaments and muscles are not shown)

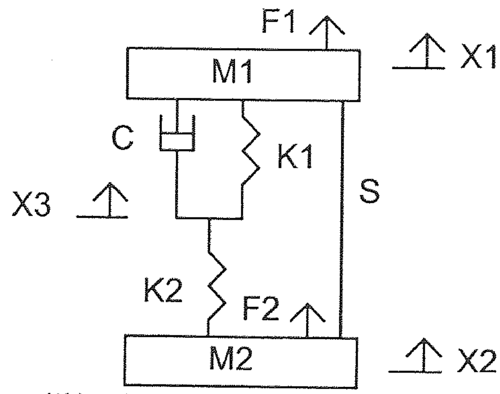


Figure (1b) Spinal Element mechanical representation

The mechanical model of this arrangement is shown in figure (1b). The element is a uniaxial element with two degrees of freedom X1, X2 (X3 is an intermediate variable which is left here only to simplify the equations representation). It has two masses (M1, M2) to represent the vertebras. The string (S) represents the ligaments and the three-element model (springs (K1, K2) and the dashpot C) is used to represent the dynamic of the disc and muscles. The forces F1, F1 represent the muscle forces and body weight.

The element equations of motion is given as

$$M_1 \cdot \ddot{X}_1 + C \cdot (\dot{X}_1 - \dot{X}_3) + K_1 \cdot (X_1 - X_3) + S(X_1 - X_2) = F_1$$

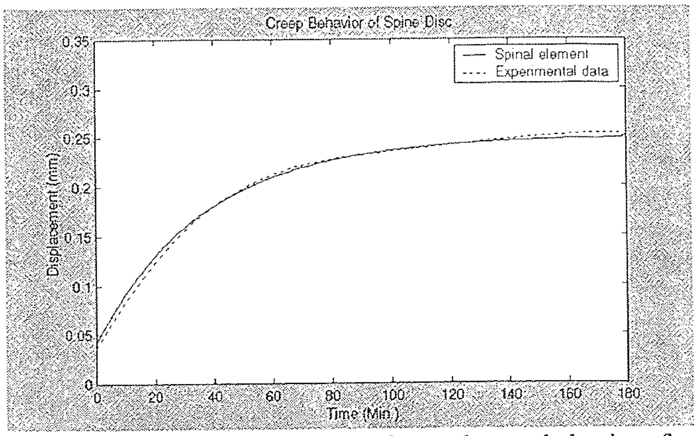
$$M_2 \cdot \ddot{X}_2 + K_2 \cdot (X_2 - X_3) + S(X_2 - X_1) = F_2$$

$$C \cdot (\dot{X}_3 - \dot{X}_1) + K_1 \cdot (X_3 - X_1) + K_2 \cdot (X_3 - X_2) = 0 \qquad (1)$$

The mass, spring and dashpot constants in equation (1) are estimated from the disc dynamics and its numerical values are given in Table (1). These values agree with the experimental results given in reference [1] as shown in Figure (2).

C (N.sec/mm)	K1 (N/mm)	K2 (N/mm)
962700	436	2530

Table(1) Numerical Values for the Three-element Model



Figure(2) Comparison with experimental creep behavior of the disc

Ligaments carry only tension loads and can be approximated by second order polynomial S(x) as given in equation (2).

$$S(x) = \begin{cases} a + bx + cx^2 & x > 0 \ (tension) \\ 0 & x < 0 \ (compression) \end{cases} \tag{2}$$

For example, for the lumbar Anterior Longitudinal Ligament (ALL) (a = 1.44, b = -34.8, c = 282.6) and for the lumbar Posterior Longitudinal Ligament (PLL) (a = 1.41, b = 106.6, c = 650.6). These values match the data given in Reference [2].

3. Spinal Motion

Being a uniaxial element, the spinal element can represent the deformation in the axial tension and compression only. We can actually represent each two vertebras with one element if we are interested in this kind of motion and still get accurate results as shown in Figure (2). Generally, we are interested in spine flexion, extension and bending motions which require rotational motion. In this case we need more than one element for each adjacent vertebras (at least two for one rotation and three for combined motion with two rotations).

4. Flexion Model

In case of single rotation, we need to get one of the vertebras to rotate relative to the other one so by adding all the resulting motion we can have the required rotation (Figure (3a)). We assumed having two spinal elements (E1, E2), with a relative displacement Δx and relaxed the rigidity condition in the same vertebra and allowed a relative motion by introducing s spring connection as shown in Figure (3b). The spring constant for this connection is a design variable and can be set from kinematic simulation data of the spine.

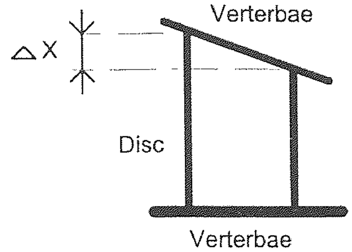


Figure (3a) actual flexion

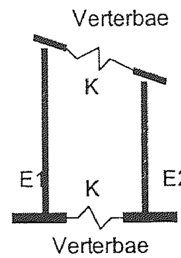


Figure (3b) flexion modeling with two spinal elements and spring constraints.

The spring forces between two spinal elements (i, j) in the direction of the spinal element motion can be calculated from the geometry and can be approximated by the following equation

$$F_{i,j} \approx K \cdot \Delta x^3$$

Adding equations (1) and (3) we get the final system of equations. The equations for element i is given by

$$M_{3i+1} \cdot \ddot{X}_{3i+1} + C_i \cdot (\dot{X}_{3i+1} - \dot{X}_{3i+3}) + K_{2i+1} \cdot (X_{3i+1} - X_{3i+3}) + S(X_{3i+1} - X_{3i+2}) + \sum_{j}^{upper\ \text{vertebrae}} F_{i,j} = F_i$$

$$M_{3i+2} \cdot \ddot{X}_{3i+2} + K_{2i+2} \cdot (X_{3i+2} - X_{3i+3}) + S(X_{3i+2} - X_{3i+1}) + \sum_{j}^{lower\ \text{vertebrae}} F_{i,j} = F_{i+1}$$

$$C_i \cdot (\dot{X}_{3i+3} - \dot{X}_{3i+1}) + K_{2i+1} \cdot (X_{3i+3} - X_{3i+1}) + K_{2i+2} \cdot (X_{3i+3} - X_{3i+2}) = 0$$

5. Lumbar L2-L3 flexion modeling

To model the L2-L3 spinal unit, we used two spinal elements as shown in Figure (3b) and assumed that L4 is fixed (X_2=X_6=F_2=F_4= 0) and L3 is allowed to rotate relatively. So the system is reduced to a two degrees of freedom system with only X_1 and X_4. The string forces in element one disappeared as it is subjected to compression force. After some simplifications, the resulting system of equations is given by

$$CM_1\ddot{\ddot{X}}_1(t)+(K_1+K_2)M_1\ddot{X}_1(t)+CK_2\dot{X}_1(t)+K_1K_2X_1(t)=$$

$$(C\frac{d}{dt}+K_1+K_2)[F_1(t)-K(X_1(t)-X_4(t))^3]$$

$$CM_3\ddot{\ddot{X}}_4(t)+(K_1+K_2)M_3\ddot{X}_4(t)+CK_2\dot{X}_4(t)+K_1K_2X_4(t)=$$

$$(C\frac{d}{dt}+K_1+K_2)[F_3(t)-K(X_4(t)-X_1(t))^3-(a+bX_4(t)+cX_4(t)^2)]$$

A comparison between the results of this model and the results given in Reference [5] for different applying moment is shown in the following figure.

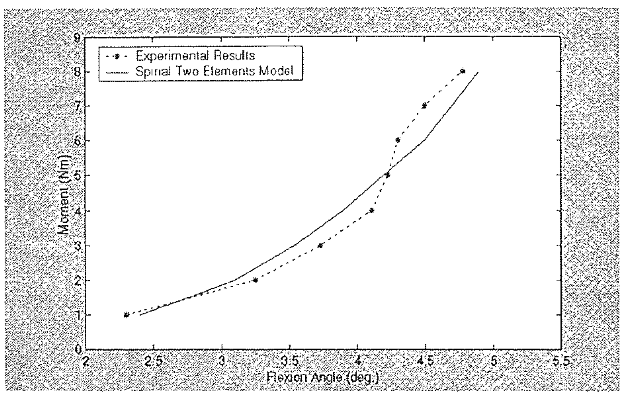


Figure (4) the relation between flexion angle and the applied moment

6. Conclusion

The spinal element introduced in this paper is a simple element with two degrees of freedom. The resulting equations are straightforward and simple to program on a computer. Spinal motions like flexion, extension and bending motions can be modeled with few elements and the comparison with experimental results shows good agreement. Finally, the accuracy of the model can be improved incrementally by adding more elements.

7. References

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